Nernst effect in high $T_c$ superconductors

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Outline

• Introduction to the Nernst effect
• Nernst effect in underdoped cuprates
• High field Nernst effect and the upper critical field $H_{c2}$
• The phase fluctuation picture of the Nernst effect
• other related experiments
• Recent controversies about the Nernst effect

The collaboration

Prof. N. Phuan Ong

Prof. Zhu-An Xu

Samples:
Yoichi Ando, CRIEPI, Tokyo
S. Uchida, Tokyo
Genda Gu, Brookhaven
Liang, Bonn and Hardy, UBC
Y. Tokura, Tokyo
Walther Nernst (1864-1941)

- Third Law of Thermodynamics: Nernst theorem
- Electrochemistry, Photochemistry, Thermochemistry
- 1920 Nobel prize in chemistry
- Theory of solutions
- Nernst Effect ...
The Nernst effect

PhD Thesis (1887) with Ettingshausen: “Electromotive forces produced by magnetism in heated metal plates”

A more general definition:
The transverse electrical field ($E_y$) produced by a longitudinal temperature gradient ($-\nabla T_x$) in a perpendicular magnetic field ($B_z$).
Schematic experimental setup

- mm-sized single crystals
- Heat sinked on sapphire substrate
- To generate a temperature gradient: thin film alloy heater glued on top of the crystal
- To measure the temperature gradient: a pair of fine-gauge thermocouples
- Voltage measurements: DC nanovoltmeter (digital multimeter with pre-amp)
Experimental setup

Joe Checkelsky thesis (2010)
The formal transport equation in normal metals

Electrical: $E$
Thermal: $-\nabla T$
Magnetic: $H$

Charge: $J_e$
Heat: $J_Q$
Spin: $J_M$

\[
J_e = \sigma E + S\sigma ( - \nabla T )
\]
\[
J_Q = \Pi \sigma E + \kappa ( - \nabla T )
\]

- This is the general electro-thermal transport equation.
- It relates the electrical and thermal current with electrical field and thermal gradient via experimental observables.
Thermoelectric transport

- Put the metal in electrical isolation: $J_e = 0$
- Longitudinal temperature gradient $-\nabla T_x$ generates a longitudinal electrical field $E_x$, which is the thermopower.
- $S = E / \nabla T_x$
Thermo-magnetic transport

• In a perpendicular magnetic field, the charge carriers will be deflected and spawn a Hall-like electrical current.

• If the two current cannot cancel out, then a transverse electrical field will be created.

• This thermomagnetic effect is the Nernst effect.
• In one-band simple metals the effect is usually very small due to the cancellation of the two currents, called the Sondheimer cancellation

• In Boltzmann equation, the Nernst signal \( e_y = E_y \nabla T = -\frac{\pi^2 k_B T}{3e} \left( \frac{\partial \theta}{\partial \epsilon} \right)_\mu \)

• proportional to the energy derivative of Hall angle evaluated at the chemical potential

• Usually small, but hard to understand exactly
In type II SC, the transverse Josephson phase slip E-field generated by the vortex motion in a longitudinal temperature gradient, is called the vortex Nernst effect.

Selective sensitivity to vortex motion!
Vortex structure and the Josephson effect in type II SC

Vortex core: $\xi$, coherence length

Josephson Equation: $2eV_J = \hbar \dot{\phi}$
$\phi = \theta_1 - \theta_2$

Vortex motion: $2eV_j = 2\pi\hbar \dot{n}_V$

$dc$ Josephson voltage proportional to phase-slip rate: $E = B \times v$
Ettingshausen effect in type II SCs

- Nernst effect: $T$ gradient $\rightarrow$ electrical field
- Ettingshausen effect: electrical field $\rightarrow$ $T$ gradient
- They are reciprocal effect
- In conventional type II SC it is much easier to do Ettingshausen, which depins the vortices by Lorentz force
Ettingshausen effect in conventional type II SCs

- Confirmed the existence and motion of vortices
- Vortex dynamics and thermodynamics
- Magnetic phase diagram

Serin et al., PRL 1967, on pure Nb
Vidal et al., PRB 1973, on PbIn
Ettingshausen effect on OPT YBCO

on optimally doped YBCO single crystals from Batlogg’s group (PRL 1990)

Upper panel:
- T evolution of resistance in fixed magnetic field

Lower panel:
- T evolution of Ettingshausen effect
- Systematic evolution with varied magnetic field
- Significant fluctuation effect (the Ettingshausen signal starts to appear at ~ 110K while Tc = 87K)
Nernst effect on optimally doped cuprates

Ettingshausen effect on optimally doped YBCO from Batlogg’s group  
PRL 1990

Nernst effect on optimally doped YBCO from Ri et al.  
PRL 1990
Nernst effect in optimally doped YBCO

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6.99} \]
\[ T_c = 92 \text{ K} \]

Temp dependence of Nernst effect

Superconducting fluctuation to 107 K
Nernst effect in optimally doped Bi-2223

Nernst effect onset temperature is around 135K, about 25K above $T_c$. 

![Graph showing Nernst effect](image)
Nernst effect in underdoped YBCO

Nernst vs. H in UD YBCO with Tc = 50K

Vortex-like signal appears at ~100K!
Nernst effect in underdoped LSCO

Nernst vs. H in underdoped LSCO

Vortex signal at 90K above $T_c$!

LSCO is tricky due to a large background Nernst effect (Wang et al., PRB 2001)
Nernst effect in underdoped Bi-2212

The Bi family is the most ideal system due to very small normal state contributions
Doping dependence of the Nernst signals

- Nernst effect on three Bi-2212 compounds with different dopings
- The fluctuation effect is more pronounced in the underdoped regime
Doping dependence of the Nernst signals

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A new crossover T scale: $T_{\text{Nernst}}$

• vortex appears at $T$ high above $T_{c0}$, especially in the underdoped regime

• vortex signal grows continuously into the SC state below $T_{c0}$
Theoretical models about the Nernst effect

• Non-superconducting theory
  exotic transport behavior in the pseudogap regime, bi-polarons, unconventional density waves, etc.

• Gaussian fluctuation theory
  enhanced amplitude fluctuations due to high Tc, short coherence length, strong anisotropy, etc.

• Phase fluctuation theory
  Kosterlitz-Thouless transition, 3D-XY model with large anisotropy

• Microscopic theory
  spin-charge separation, staggered flux, phase string/spinon vortices, etc.
The importance of the phase degree of freedom

Complex SC order parameter:
\[ \psi(r) = |\psi(r)| e^{i\theta(r)} \]

| SC state: | \(|\psi(r)| \neq 0 + \text{long range phase coherence} \) |
| Non-SC state: | \(|\psi(r)| = 0\), normal state of conventional SC |
| | \(|\psi(r)| \neq 0\), no long range phase coherence |

Nernst effect in cuprates:
vortex-like excitations above \( T_c \)

Finite Cooper pairing amplitude + short range phase correlation

In high \( T_c \) cuprates, the phase transition at \( T_c \) is caused by the loss of long range phase coherence rather than the vanishing of pairing amplitude
\[ \nabla \theta \]

\[ \nu_s = \frac{\hbar \nabla \theta}{2m^*} \]

A twist of phase $\nabla \theta$  \quad \xrightarrow{\text{Cooper pair motion}} \quad \nu_s = \frac{\hbar \nabla \theta}{2m^*}

Kinetic energy cost \quad \xrightarrow{\text{Cooper pair motion}} \quad \varepsilon = \frac{1}{2} n_s.2m^*.\nu_s^2 \propto n_s (\nabla \theta)^2

- Conventional SC: $n_s$ is very large, strong phase coherence
- High $T_c$ cuprates: $n_s$ is very small, strong phase fluctuations

At $T_c$: unbinding of thermally excited vortices destroy long range phase coherence, but not local pairing amplitude
Strong phase fluctuation and the phase diagram of high $T_c$

Doped Mott insulator: $n_s$ is proportional to $x$
µSR (Uemura), microwave (Bonn & Hardy)

$\Delta$: gap amplitude (ARPES, STM) pairing strength decreases with $x$

$T_{MF}$: mean-field pair formation $T$
$T_\theta$: phase stiffness temperature

$T_c$ phase boundary: $T_c = \min (T_{MF}, T_\theta)$

For $T < T_{onset}$:
Phase incoherence condensate
Vortex-like excitations are detectable by Nernst effect

Are these signals really from vortices ???

Nernst effect of vortices and normal carriers

Vortex Nernst signal
“mountain” like features:
  Jump, peak, linear decrease

Normal carrier Nernst signal
Linear function of H,
Hall channel response.
Motivations for high field Nernst experiments

H-T phase diagram of a conventional type II and cuprates

- What is the properties of the vortex liquid state?
- Where is the upper critical field $H_{c2}$?
- What’s the $T$ and $x$ dependence of $H_{c2}$?
Hybrid Magnet: 45 tesla

Resistive Magnet: 33 tesla
High field Nernst effect up to 45 tesla

- “mountain”-shaped vortex Nernst effect on OPT LSCO up to 45 T

- Continuous evolution across $T_c$
High field Nernst effect in Bi-2201

- Continuous evolution across $T_c$
- strong vortex features above $T_c$
“Mountain”-like Nernst signal at $T$ above $T_c$ in UD cuprates

- stronger vortex signals above $T_c$ in underdoped cuprates
- Nernst signal at $T_c$ has one of the largest values
Evolution of the magnitude of the Nernst signal with doping

Doping dependence of high field Nernst effect in LSCO

Strong contrast between UD and OVD LSCO
Define $H_{c2}$ from high-field Nernst measurements

Upper critical field $H_{c2}$: pairing strength and coherence length

$T=1.5\text{K}$, PbIn

Linear extrapolation of the Nernst effect to define $H_{c2}$ in optimally doped LSCO

F. Vidal, Ettingshausen effect
Flux-flow resistivity is a bad diagnostics for $H_{c2}$

- MR shows broadening without a well-defined $T$ or $H$ scale
- above the MR-defined $H_{c2}$ there is still strong vortex Nernst signal
High field Nernst effect on Optimally doped Bi-2201

- Convergent behavior of Nernst traces
- Constant $H_{c2}$ for a wide range of $T$
- finite $H_{c2}$ for $T > T_c$

• in convention SC:

\[ H_{c2}(T) \propto (1-t) \text{ for } T < T_c \]

it terminates at \( T_c \)

• in cuprates:

\[ H_{c2}(T) = \text{const. for } T < T_c \]

it is finite for \( T \geq T_c \)

both \( H_{c2} \) and \( \Delta \) measure the Cooper pairing strength

• From \( H_{c2} \): magnetic length \( \xi_H \)

\[ H_{c2} = \Phi_0 / 2\pi \xi_H^2 \]

• From \( \Delta \): Pippard length \( \xi_P \)

\[ \xi_P = h\nu_F / \alpha\Delta \]

in conventional type II, \( \xi_H = \xi_P \)

\[ : H_{c2} \propto \Delta^2 \]
Nernst traces of 3 Bi-2212 with different dopings

Underdoped cuprate has broader features

Underdoped cuprate has larger $H_{c2}$ than opt and overdoped!
• Successively larger field is needed to suppress Nernst signal
• Optimally doped Bi-2201 sample as template for comparison
Scaling properties of the Nernst signals

Template: opt doped Bi-2201
\( H_{c2} = 50 \text{ T} \)

Overdoped Bi-2212, \( T_c = 65 \text{ K} \)
\( H_{c2} = 50 \text{ T} \)

Opt doped Bi-2212, \( T_c = 90 \text{ K} \)
\( H_{c2} = 70 \text{ T} \)

Underdoped Bi-2212, \( T_c = 50 \text{ K} \)
\( H_{c2} = 140 \text{ T} \)

\( H_{c2} \) increases as \( x \) lowers
Doping dependence of $H_{c2}$

- $H_{c2}$ can be determined by the scaling analysis
- $H_{c2}$ increases in the underdoped regime
- Similar trend as the energy gap
- Stronger pairing strength in the UD regime of the cuprate

Doping dependence of coherence length $\xi$

- Convert both $H_{c2}$ and $\Delta$ to coherence length $\xi$
- They agree quantitatively
- Vortex core size measured by STM, SH Pan et al., $\xi \sim 22 \text{ Å for } x = 0.16$
- As $x$ decreases, $\xi$ becomes smaller.
- In the underdoped regime: very strongly and tightly bounded Cooper pairs.
Nernst effect in electron doped cuprates

- large contribution to the Nernst by normal carriers
- two bands, no Sondheimer cancellation. ARPES (ZX Shen group)
- different characteristics enable us to separate the vortex signal
Vortex Nernst signal and the $H$-$T$ phase diagram of NCCO

- Vortex Nernst effect is completely different from hole doped ones
  - no vortex signal above $T_c$
  - conventional $H_{c2}$-$T$ phase line
  - no pseudogap phase?
Summary of Nernst results

1. Vortex-like excitations high above $T_c$ in underdoped cuprates

2. The upper critical field $H_{c2}$ derived from Nernst effect
   
   a. $H_{c2}$ is weakly $T$ dependent for $T < T_c$, it is finite for $T \geq T_c$

   b. underdoped cuprates have larger $H_{c2}$ and pairing strength

3. Lack of strong fluctuation and conventional phase diagram in NCCO

Other experiments that are consistent the Nernst results

**Transport:**
- Terahertz (Orenstein)
- Magnetoresistance (Hussey)

**Thermodynamics:**
- Thermal expansion (Meingaist)
- Diamagnetism (Wang and Ong)

**Spectroscopy:**
- ARPES (Campuzano)
- STM (Yazdani and Davis)
Fluctuation induced diamagnetism

Magnetization in Abrikosov state

The upper critical field $H_{c2}$ of cuprates from the M-H measurements?
Cantilever magnetometer for diamagnetism measurements

Cantilever magnetometer:
\[ \tau = m \times B = mB \sin \theta \]

Micro-fabricated single crystal silicon, cantilever magnetometer

- beam thickness \( \sim 10 \, \mu \text{m} \)
- Capacitive detection of deflection
- Sensitivity: \( \sim 5 \times 10^{-9} \, \text{emu at 10 tesla} \), easy to go high field
cantilever magnetometer for diamagnetism measurements

cantilever magnetometer setup at Tsinghua
Torque measurements on opt doped Bi-2212, $T_c = 90$ K
Torque measurements on underdoped Bi-2212, $T_c = 50$ K:

- $T >> T_c$:
  $$\tau \sim H^2, \ m \propto H,$$
  weakly $T$-dependent paramagnetic background

- $T \leq 120$ K:
  onset of negative contribution

- $T \sim T_c$:
  significant negative $\tau$ value

- $T < T_c$:
  $\tau$ decreases very rapidly
Torque measurements on underdoped Bi-2212, $T_c = 50$ K

- strongly enhanced diamagnetic signals at $T > T_c$ in UD cuprates,
- consistent with the Nernst effect
- vortex liquid state above $T_c$
Both the T and H dependence of the diamagnetic signal closely tracts the Nernst effect in the fluctuation regime

- vortex Nernst effect: transport, E-field by vortex motion
- diamagnetism: thermodynamic, diamagnetic current by vortices

Vortex liquid state extends to T above $T_c$!
- Same trend in all Bi-2212
- agreement of Nernst and diamagnetism

STM measurements of the local pair formation

Visualizing pair formation on the atomic scale in the high-$T_c$ superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta$

Kenjiro K. Gomes$^1$, Abhay N. Pasupathy$^1$, Aakash Pushp$^1$, Shimpei Ono$^2$, Yoichi Ando$^2$ & Ali Yazdani$^1$

Variable temperature STM can directly probe local energy gap, and track the same location over a large temperature range.
- Optically doped Bi-2212 with $T_c = 93K$
- Energy gap survives to $T \sim 140K$
STM measurements of QPI

Spectroscopic Fingerprint of Phase-Incoherent Superconductivity in the Underdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

Jhinwhan Lee,¹ K. Fujita,¹,² A. R. Schmidt,¹ Chung Koo Kim,¹ H. Eisaki,³ S. Uchida,² J. C. Davis¹,⁴

Science (2009)

- UD Bi-2212 with Tc = 37K
- STM QPI probes the interference of the particle-hole symmetric Bogoliubov qp
- There is no change of the octet QPI pattern at Tc
- It survives at least to 1.5Tc
- The state above Tc has phase-incoherent superconducting pairing
Recent controversy regarding the Nernst results

- Eu or Nd doped LSCO with or without stripes
- The Nernst effect has an increase above the stripe formation temperature
- For samples without stripes there is no such enhancement
- There is a second increase at low T due to superconducting fluctuations
- Conclusion: stripe order can enhance the qp Nernst effect

Recent controversy regarding the Nernst results

- We were well aware of the qp background problem in LSCO, and spent tremendous amount of effort to subtract the background

- Only when the Nernst signal deviate from the qp background do we define the onset of vortex signal (PRB 2001)
Recent controversy regarding the Nernst results

Nernst effect and the loss of superconducting phase-coherence in underdoped $\text{YBa}_2\text{Cu}_3\text{O}_y$.

Yayu Wang, Z. A. Xu$^1$ and N. P. Ong
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544.
(November 10, 2009)
Recent controversy regarding the Nernst results

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**LETTERS**

**Broken rotational symmetry in the pseudogap phase of a high-$T_c$ superconductor**

R. Daou¹, J. Chang¹, David LeBoeuf¹, Olivier Cyr-Choinière¹, Francis Laliberté¹, Nicolas Doiron-Leyraud¹, B. J. Ramshaw², Ruixing Liang², D. A. Bonn²,³, W. N. Hardy²,³ & Louis Taillefer¹,³

Princeton University PhD thesis (2004), P131, Yayu Wang
• The Nernst results of Ong’s group and Taillefer’s group are similar, the controversy mainly lies in the interpretation

What Taillefer’s Nernst results have proved:

• In stripe phase LSCO and YBCO near $x = 6.67$, there is a large qp Nernst effect caused by the formation of stripes or other density wave order
• There are exactly the regimes that we avoided for the vortex picture

What Taillefer’s results have not proved:

• When and how the fluctuation induced Nernst effect start to appear
• Especially, they haven’t shown any results on the Bi compounds, which is the cleanest system to study the Nernst effect, and many other results (ARPES, STM, Optics, Diamagnetism) are available

The debate is not an independent one, and is part of the bigger debate regarding the nature of the pseudogap phase
Thanks for your attention!
• $M \sim \ln H$ for a wide range of field

• $H_{c2} \sim 60$ tesla for OD Bi-2212, agree with Nernst

• $H_{c2}$ is weakly $T$-dependent
High field magnetization on cuprates with a wide range of doping
Relationship between $H_{c2}$ and $T_{\text{onset}}$

- $k_B T_{\text{onset}} = g \mu H_{c2}$
- $g \sim 2.1$
Summary of results

Vortex Nernst effect and enhanced diamagnetism in cuprates

- onset of vortex signal at $T$ high above $T_c$: strong phase fluctuations in the pseudogap phase
- anomalous high field phase diagram: unconventional temperature and doping dependence of $H_{c2}$

References:


Summary of results

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  bi-polarons, unconventional density waves, etc.

• Gaussian fluctuation theory
  enhanced amplitude fluctuations due to high Tc,
  short coherence length, strong anisotropy, etc.

• Phase fluctuation theory
  Kosterlitz-Thouless transition, 3D-XY model with large anisotropy

• Microscopic theory
  spin-charge locking, staggered flux, phase string/spinon vortices,
  vortices with multi flavors, etc.
In low $T_c$ superconductors, droplets of superfluid exist above $T_c$.

\[
\chi^{3D} = \left(\frac{1}{6\pi}\right) \left(\frac{e}{\hbar}\right)^2 k_B T_c \xi_{GL} \sim -10^{-7} (t-1)^{-1/2}
\]

\[
\chi^{2D} = \left(\frac{1}{3\pi}\right) \left(\frac{e}{\hbar}\right)^2 k_B T_c \xi_{GL}^2/d \sim (t-1)^{-1}
\]

This is $\sim 50$ times smaller than observed in underdoped Bi 2212!

Scaling analysis:
For $T' < T_c$, $M'$ is a constant

For UD Bi-2212,
$M' \sim$ constant at $T' = 46$ K for $H < 4$ T
It fails at high field

Some non-Gaussian theory is needed at $T > T_c$
Vortex-liquid state, Kosterlitz-Thouless transition