The Vortex Matter
In High-Temperature
Superconductors

Yosi Yeshurun
Bar-Ilan University
Ramat-Gan
Israel
The Vortex Matter In High-Temperature Superconductors

Vortices in superconductors – dynamics and thermodynamics

Global and local magnetic measurements
Conductors

Energy is carried by electrons

- Resistance
  - A result of interaction with
    - Other electrons
    - Vibrating atoms
    - Impurities

- Energy dissipation
High-Temperature Superconductors

conductors

Superconductors

Temperature
How to reach low temperatures?

Cooling gases down:
Compress/expand the gas adiabatically.
Part of the gas liquefies and collected.

- Oxygen, $\text{O}_2$: 90K
- Nitrogen, $\text{N}_2$: 77K
- Hydrogen, $\text{H}_2$: 20K

1908: H. Kamerlingh Onnes

- Helium, $\text{He}_4$: 2 – 4 K

1913: Nobel Prize
Discovery of Superconductivity

Heike Kamerlingh Onnes

- 1908 - liquefied helium (~4 K = - 269°C)
- 1911 - investigated low temperature resistance of mercury
High-Temperature Superconductors

conductors

Superconductors

Temperatures

High-temperatures
HEATING UP

Highest known superconducting temperatures

Tentative indications at 240K (-28°F)

Feb. 1987
98K (-283°F)

Nitrogen liquefies at 77K (-320°F)

April 1986
35K (-397°F)

1911 Mercury

1940 Niobium compounds

1960

1980

### Periodic Table of the Elements

<table>
<thead>
<tr>
<th>Period</th>
<th>Group</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IA</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>IIA</td>
<td>Li, Be</td>
</tr>
<tr>
<td>3</td>
<td>IIIA</td>
<td>Na, Mg</td>
</tr>
<tr>
<td>4</td>
<td>IVA</td>
<td>Al, Si</td>
</tr>
<tr>
<td>5</td>
<td>IVA</td>
<td>P, S, Cl</td>
</tr>
<tr>
<td>6</td>
<td>VA</td>
<td>Ar</td>
</tr>
<tr>
<td>7</td>
<td>VIA</td>
<td>K, Ca</td>
</tr>
<tr>
<td>8</td>
<td>VIIB</td>
<td>Sc, Ti</td>
</tr>
<tr>
<td>9</td>
<td>VIIB</td>
<td>V, Cr</td>
</tr>
<tr>
<td>10</td>
<td>VIIB</td>
<td>Mn, Fe</td>
</tr>
<tr>
<td>11</td>
<td>VIIB</td>
<td>Co, Ni</td>
</tr>
<tr>
<td>12</td>
<td>VIIB</td>
<td>Cu, Zn</td>
</tr>
<tr>
<td>13</td>
<td>VIIB</td>
<td>Ga, Ge</td>
</tr>
<tr>
<td>14</td>
<td>VIIB</td>
<td>As, Se</td>
</tr>
<tr>
<td>15</td>
<td>VIIB</td>
<td>Br, Kr</td>
</tr>
<tr>
<td>16</td>
<td>VIA</td>
<td>Rb, Sr</td>
</tr>
<tr>
<td>17</td>
<td>VIA</td>
<td>Y, Zr</td>
</tr>
<tr>
<td>18</td>
<td>VIA</td>
<td>Nb, Mo</td>
</tr>
<tr>
<td>19</td>
<td>VIA</td>
<td>Tc, Ru</td>
</tr>
<tr>
<td>20</td>
<td>VIA</td>
<td>Rh, Pd</td>
</tr>
<tr>
<td>21</td>
<td>VIA</td>
<td>Ag, Cd</td>
</tr>
<tr>
<td>22</td>
<td>VIA</td>
<td>In, Sn</td>
</tr>
<tr>
<td>23</td>
<td>VIA</td>
<td>Sb, Te</td>
</tr>
<tr>
<td>24</td>
<td>VIA</td>
<td>I, Xe</td>
</tr>
<tr>
<td>25</td>
<td>VIA</td>
<td>Cs, Ba</td>
</tr>
<tr>
<td>26</td>
<td>VIA</td>
<td>Fr, Ra</td>
</tr>
</tbody>
</table>

- **Lanthanide Series**: Ce, Pr, Nd, Pm, Sm, Eu, Gd, Tb, Dy, Ho, Er, Tm, Yb, Lu
- **Actinide Series**: Th, Pa, U, Np, Pu, Am, Cm, Bk, Cf, Es, Fm, Md, No, Lr
HEATING UP

Highest known superconducting temperatures

Tentative indications at 240K (-28°F)

Feb. 1987
98K (-283°F)

Nitrogen liquefies at
77K (-320°F)

New oxide compounds

April 1986
35K (-397°F)

1911 Mercury

Niobium compounds

Copyright (c) 1980
**HEATING UP**

Highest known superconducting temperatures

- **Tentative indications at 240K (-28°F)**
- **Feb. 1987 98K (-283°F)**
- **Nitrogen liquefies at 77K (-320°F)**
- **New oxide compounds**
- **April 1986 35K (-397°F)**

1911 Mercury

Niobium compounds

Copyright © 1980

[Copyright © 1940 1960 1980 Absolute zero (-460°F)]
<table>
<thead>
<tr>
<th>Material ((T_c))</th>
<th>Discovery</th>
<th>Rapid follow-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\text{La, Ba})_2\text{CuO}_4) (30 K)</td>
<td>IBM (Zurich)</td>
<td>U. Tokyo, U. Houston, AT&amp;T, Bellcore</td>
</tr>
</tbody>
</table>

Hg-Ba-Ca-Cu-O at 136 K
164 K under pressure
What is so SUPER about superconductors?

Large critical current (the maximum current that can be carried by the superconductor without energy dissipation)
What is so SUPER about superconductors?

**Large critical current** (the maximum current that can be carried by the superconductor without energy dissipation)

1. High current applications
- Transferring energy without loss
Energy transfer
1. **High current applications**

- Transferring energy without loss
- Superconducting Magnetic Energy Storage (SMES)
Superconducting Magnetic Energy Storage (SMES)

Motivation: (1) Storing energy  
(2) Improving energy quality

Other solutions:  
• Fly Wheels  
• Batteries  
• Capacitors
Characteristic Discharge Times of Different Storage Devices

- Capacitor
- SMES
- Flywheel
- Battery

Time Scale: 1 ms, 1 s, 1000 s
1. **High current applications**
   - Transferring energy without loss
   - Superconducting Magnetic Energy Storage (SMES)
   - Fault Current Limiter (FCL)
Fault Current Limiter (FCL)

Motivation: Limit the utility current

**Required features:**
- Low impedance at normal operation
- Fast response (less than half a cycle)
- Self recovery

**Methodology:**
- Resistive
- Inductive
Saturated Cores FCL

Utilizing the non-linear magnetic characteristic of iron core

Each half-cycle is treated separately
Bar-Ilan’s system is the most effective FCL for mid and high voltage, and is much more compact and lighter than any other alternative. Operating at 30–40 K; closed cycle refrigeration.
Most Efficient FCL Solution

- Invisible during normal grid operation
- Passive, self-triggered, immediate reaction
- Unlimited holding time - until breakers trip
- Zero recovery time to ensure fault protection at all times
- Impedance-tuning to control the level of a current supply
- Fail-safe
- Compact, light, economical design
1. **High current applications**
   - Transferring energy without loss
   - Superconducting Magnetic Energy Storage (SMES)
   - Fault Current Limiter (FCL)
   - Medical applications - Magnetic Resonance Imaging (MRI)
Medical applications:
Magnetic Resonance Imaging (MRI)
MRI - the most expensive equipment in the hospital

At $2 million, the most expensive equipment in the hospital...
Magnetic Resonance Imaging

**Advantages:**
- Excellent / flexible contrast
- Non-invasive
- No ionizing radiation
- Arbitrary scan plane

**Challenges:**
- New contrast mechanisms
- Faster imaging
What makes superconductors SUPER?

- **Insulator**: In materials with extremely high resistance, such as rubber or glass, electrons are tightly bound to atoms and cannot be jostled loose to sustain a flow of current.

- **Conductor**: In materials with lower resistance, some electrons are loosely bound and form a current when voltage is applied. Resistance is a measure of the energy lost in the form of heat from electron collisions.

- **Superconductor**: When materials become superconductive, all resistance disappears because electrons are bound into pairs, which move in step with each other, avoiding collisions. Current flows with no energy loss.
BCS Theory of Superconductivity

By John Bardeen, Leon Cooper, and Robert Schrieffer

A key conceptual element in this theory is the *pairing of electrons* close to the Fermi level into Cooper pairs through interaction with the crystal lattice.
Cooper Pairs
Animation of Cooper pairs:

Source: superconductors.org and Ian Grant
What is a superconductor?

1) **Zero electrical resistivity**

   NO!!! Insufficient characterization!!!

   1. May be a perfect conductor
   2. A superconductor may exhibit non-zero resistance (due to vortex motion)

2) **The Meissner effect**
Meissner Effect

1933: Walther Meissner & Robert Ochsenfeld

$T < T_c$: external magnetic field is perfectly expelled from the interior of a superconductor

As a result: (1) Diamagnetism (2) Mag. Levitation

http://www.jsf.or.jp/sln/aurora_e/step2.html
**Perfect Diamagnetism**

\[ \chi_m = -1 \]

- **Means:**

\[ B = \mu_0(H + M) \]
\[ B = \mu_0(H + \chi_m H) \]
\[ B = 0 \]

**Normal Metal**  
**Superconductor**

Flux is excluded from the bulk by supercurrents flowing at the surface to a penetration depth \( \lambda \sim 200-500 \text{ nm} \)
The Meissner Effect
Magnetic Levitation
TRANSPORTATION
Magnetic Levitation (MagLev)
Meissner (and Ochsenfeld) Effect [1933]

- T<T<sub>c</sub>: external magnetic field is perfectly expelled from the interior of a superconductor

- Strong external magnetic fields: Shielding is partial

- Stronger magnetic fields destroy superconductivity

http://www.jsf.or.jp/sln/aurora_e/step2.html
H-T diagram
of conventional superconductors
Thermodynamic H-T phase diagram of conventional superconductors
Thermodynamic H-T phase-diagram of superconductors

1) Zero electrical resistivity
   - Electrical current in a superconducting ring continues indefinitely

2) The Meissner effect
   - Magnetic field inside a bulk sample is zero -- current flows to shield the external field (the induced field opposes the applied field)
   - The material is strongly diamagnetic as a result.
   - Magnetic levitation
Experimental confirmation of the Abrikosov Lattice

- Magnetic Decoration
- Magneto-Optical Imaging
- Low-Angle Neutron Scattering
Vortex in type II superconductor

Magnetic decoration

\[ \Phi_0 = \frac{hc}{2e} \]
Magneto-optical setup

- PC MATLAB
- CCD camera Hamamatsu C4880-80
- Green Filter
- Polarizer
- Analyser 360°
- Microscope Leica DMRM
- Bi:YIG indicator
- Cryostat Oxford
- Liquid He
- Heater
- Coil Lipman PS
- Sample
Vortices in type II superconductor

Flux quantum $\Phi_0 = \frac{hc}{2e}$

Magneto-optical image of a quasi-Abrikosov (quasi-ordered) vortex state [Johansen et al.]
Vortices in type II superconductor

Motion of vortices results in:

• Energy dissipation
• Irreversible phenomena
How to prevent/minimize energy dissipation?

Answer: Pin vortices by, e.g. defects

Pinned vortices → Magnetic irreversibility → Larger critical currents

Lorentz force ($\sim B \times I$) moves the vortices at all fields. As a result: A superconductor with resistance that increases with $H$. 
Max current that can be carried by the SC without dissipation.

\( \text{Width} \sim j \)

\( j \) reflects vortex pinning in defects and hence a disorder in the vortex lattice.
Flux penetration

MO images

sample

Bean Profile

$J \sim dB/dx$
Vortex instabilities and pattern formation

- **Niobium film**: Duran et al., 1995

- **MgB$_2$ film**: Johansen et al., 2002.

- **LaSCO film**: Y.Y. (unpublished)

- **YBCO film**: Wijngaarden et al., 1999.

- **BSCCO crystals**: Barness, Y.Y. et al. (2008)

Thermo-magnetic instability

vortex motion dissipates energy $J \cdot E$

$E \sim dB/dt$

motion

positive feedback

Unstable if loop gain > 1

more vortices move

local temperature increases

easier for vortices to overcome pinning barriers

$+kT$

After Galperin et al.
Novel vortex instabilities
[NOT thermomagnetic mechanism]

Found only in the proximity of vortex phase transition line
Magnetic phase diagram of conventional superconductors.
Thermodynamic vortex phase diagram in BSCCO

Disordered solid
high j

Quasi-ordered solid
low j

liquid

Low-angle neutron scattering
(Cubit et al., Nature)
Magneto-optical measurements
(Johansen et al.)
Manifestation of the vortex order-disorder phase transition in magnetic measurements

Second Magnetization Peak (SMP)

Width $\sim j$

Khaykovich et al., PRL 76 (1996).
Problems in identifying the solid-solid transition

- The SMP exhibits dynamic behavior:
  - SMP is absent at short times
  - Onset and peak shift with time

- As temperature is lowered, the second magnetization peak is gradually smeared, until below a certain temperature it disappears altogether
Termination of transition-line at low temperature?
Vortex instabilities near vortex phase transitions
Vortex injection through the sample edges contamination


Disorder is induced by inhomogeneous surface barriers

Transient disordered vortex state
Evidence for Transient Disordered Vortex States (TDVS)

Time evolution of induction profiles at a constant applied field:

- $t = 0.04 \text{ s}$
- $t = 29 \text{ s}$

Evidence for Transient Disordered Vortex States (TDVS)

High $j$ phase disappears with time transient disordered vortex state
Evaluation of \( \tau(B,T) \) = \( \frac{\partial \tau}{\partial B} \) ext 1 B dH dt

Field Seep Up:

Field Seep Down:

Relaxation at constant \( H_{\text{ext}} \)

Direct measurement

\[
\left( \frac{\partial \tau}{\partial B} \right)_{B=B_0} = \frac{1}{dH_{\text{ext}}/dt}
\]

\[
\tau(B_{f0}) = \frac{B_{od} - B_{f0}}{dH_{\text{ext}}/dt}
\]

- \( 23 \text{ K} \)
- annealing experiment
- field sweep down
- field sweep up

\[ \begin{align*}
\text{B [G]} & \quad 200 \quad 250 \quad 300 \quad 350 \quad 400 \\
\tau [s] & \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5
\end{align*} \]
As $T$ decreases, $\tau(B)$ increases.

Lifetime of TDVS - $\tau(B,T)$

- $T = 21$ K
- $T = 23$ K
- $T = 25$ K
- $T = 27$ K
- $T = 30$ K

The graph shows the lifetime $\tau$ as a function of $B/B_{od}$ for different temperatures $T$. The lifetime increases with decreasing temperature.
artifact

Transition masked by TDVS

Termination of the transition line at low temperatures
Transient Disordered Vortex States are injected through inhomogeneous surface barriers during field increase.
Vortex instabilities near vortex phase transitions

• Transient disordered vortex state (in a vortex ordered phase)

• Spontaneous generation of vortex oscillations in $B(x,t)$

“Flux waves”
Disordered solid

Quasi-ordered solid

Transient states

Quasi-ordered solid

high j

low j

liquid
MO images at 24 K after application of 430 Oe

0 ms

left edge

right edge
Induction profiles

zoom on left side of the sample (waves)
Temporal oscillations

Hall-probe images (23 K)

J. Moore et al.
Imperial College, London

'Waves' are clearly related to order-disorder phase transition
Experimental conditions for the appearance of flux waves

1. Proximity to the vortex order-disorder phase transition line

2. A nearly flat induction profile (low-j phase)
Theoretical analysis

Working hypothesis: Origin of flux waves in diffusion processes in the vortex matter

Flux waves belong to the general category of spatiotemporal pattern formation (For a review, see Cross and Hohenberg, Rev. Mod. Phys. (1993))

Two COUPLED Nonlinear Diffusion Equations
The Landau-Khalatnikov equation is given by:

$$\frac{\partial \Psi}{\partial t} = -\Gamma \frac{\delta F}{\delta \Psi}$$

Free energy:

$$F = \frac{1}{2} \int \left[ D (\partial \Psi)^2 - a \Psi^2 + \gamma_0 \Psi^4 / 2 \right] d^3 r$$

Landau coefficients for the field-driven vortex phase transition:

$$\alpha = \alpha_0 (1 - B / B_{od}), \quad D, \quad \gamma_0$$

Order to disorder:

$$\text{high } J_c[\Psi] \quad \longleftrightarrow \quad \Psi = 0$$

Disorder to order:

$$\text{low } J_c[\Psi] \quad \longleftrightarrow \quad \Psi = \Psi_0(B)$$

$\Psi$ is defined as the Fourier component of the vortex density at the minimal vector of the reciprocal lattice.
2nd diffusion equation

• Relaxation of magnetic induction

\[ \frac{\partial B}{\partial t} = -c \frac{\partial E}{\partial x} \]

\[ \frac{\partial B}{\partial t} = \frac{c^2}{4\pi} \frac{\partial}{\partial x} \left[ D_f \frac{\partial B}{\partial x} \right] \]  \hspace{1cm} (2)

- **E** = \( R_F J \exp(-U/kT) \)  
  electric field

- **R_F** = \( R_n (B/B_{c2}) \)  
  flux flow resistivity

- **U**  
  pinning potential

- **U** = \( U_0 \ln(J_c/J) \)  
  U(J) logarithmic

- **J_c**  
  critical current density

- **D_f**  
  diffusion coefficient for flux creep

- **\sigma**  
  \( \frac{U_0}{T} \)
Coupling between the two nonlinear diffusion equations via \( J_c [\Psi] \)

Linear Stability Analysis predicts possibility for oscillatory instability characterized by a period and wavelength in accordance with the experimental results.
Scenario for 'waves':

- **Injection:** \( \Psi=0 \)
- **Left:** fast annealing
  - \( \Psi > 0 \)
  - lower \( J_c \) \( \rightarrow \) larger \( D_f \)
  - fast relaxation

- **Right:** \( \Psi = 0 \)
  - high \( J_c \) \( \rightarrow \) low \( D_f \)
  - slow relaxation
    - (regular creep)

**Gradient in \( D_f \):**
- accumulation of vortices

- higher \( B \) \( \rightarrow \) lower \( \Psi \)
  - higher \( J_c \) \( \rightarrow \) lower \( D_f \)
  - accumulation stops

**Inverted slope:**
- flux moves to lower \( B \) \( \rightarrow \) \( B \) decreases

**Equations:**
- (Eq. 1)
- (Eq. 2)
Summary of spontaneous oscillations phenomena

Oscillations in time

Oscillations in space

Observed near the vortex order-disorder phase transition

Two coupled nonlinear diffusion equations $\rightarrow$ instability

Instabilities associated with proximity to phase transition NOT with thermomagnetic processes
Summary and Conclusions

High-temperature superconductors

Fascinating fundamental physics

Applications are around the corner